Lösung zu P17 A1.7.9

oBdA $p>q$

Lös: $\forall\ x\in\mathbb{R}$ gilt $(1+x)^p(1+x)^q=(1+x)^{p+q}$ und

$$(1+x)^p(1+x)^q=\left[\sum_{j=0}^{p}\binom{p}{j}x^j\right]\left[\sum_{k=0}^{q}\binom{q}{k}x^k\right]=(1+x)^{p+q}=\left[\sum_{v=0}^{p+q}\binom{p+q}{v}x^v\right]$$

$(1+x)^p(1+x)^q=$

$$\left(\binom{p}{0}x^0+\binom{p}{1}x^1+\binom{p}{2}x^2+\binom{p}{3}x^3+\dots\binom{p}{p}x^p\right)+\left(\binom{q}{0}x^0+\binom{q}{1}x^1+\binom{q}{2}x^2+\binom{q}{3}x^3+\dots\binom{q}{q}x^q\right)=$$

$$\binom{p}{0}\binom{q}{0}x^0+\binom{p}{0}\binom{q}{1-0}x^1+\binom{p}{0}\binom{q}{2-0}x^2+\binom{p}{0}\binom{q}{3}x^3+\dots\quad\binom{p}{0}\binom{q}{q}x^q+$$

$$\binom{p}{1}\binom{q}{1-1}x^1+\binom{p}{1}\binom{q}{2-1}x^2+\binom{p}{1}\binom{q}{2}x^3+\binom{p}{1}\binom{q}{3}x^4+\dots\quad\binom{p}{1}\binom{q}{q-1}x^q+\binom{p}{1}\binom{q}{q}x^{q+1}+$$

$$\binom{p}{2}\binom{q}{2-2}x^2+\binom{p}{2}\binom{q}{1}x^3+\binom{p}{2}\binom{q}{2}x^4\dots\quad+\binom{p}{2}\binom{q}{q-1}x^{q+1}+\binom{p}{2}\binom{q}{q}x^{q+2}+\dots$$

$$\dots\qquad\dots\qquad\dots$$

$$\binom{p}{p}\binom{q}{q}x^{q+p}$$

$$=\sum_{v=0}^{p+q}x^v\sum_{j=0}^{v}\binom{p}{j}\binom{q}{v-j}$$ und Koeffizientenvergl.